Application of F-index on bipolar fuzzy graphs

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Abstract.

Several applications exist for various kinds of topological index of graphs, and several results are accessible for crisp graphs. However, we find many situations in practical applications that cannot be modeled using crisp graphs. To handle such a situation, we need those topological indices defined in a bipolar fuzzy graph in this article, F-index for bipolar fuzzy graphs is introduced and some bounds on F-index are provided for several bipolar fuzzy graphs such as path, star, cycle, complete bipolar fuzzy graph, etc. In this article, we get the maximal BFG with respect to the F-index given the vertex set Finally, we present on Application of F-index to find the best health center in providing medical services.

Keyword: bipolar fuzzy graph, topological indices, Forgotten Topological Index, Forgotten Topological Index for bipolar fuzzy graph.

1. Introduction

1.1. Research background. Many researchers are working on topological indices (TIs) these days due to various applications of fuzzy graph theory. Fuzzy set (FS) were initially proposed in 1965 by Zadeh [40]. This is what prompted Rosenfeld to define the fuzzy graph (FG) in 1975 [27]. At the same time, Rashmanlou and et al studied on vague graphs [26] and product of interval-valued fuzzy graphs and degree [25]. Additionally, the degree of a vertex in a FG, as well as strong degree and strong neighbor of a FG, are covered in [20].Quantitative Structure–Property Relationship Analysis in Molecular Graphs of Some Anticancer Drugs with Temperature Indices Approach was given Xiaolong Shi , Ruiqi Cai and et al [30] . Fuzzy graph theory is discussed in further depth in [14, 15, 16, 19, 21, 28]. Additionally and in 1994, Zhang [35,37] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Zhang [36] came up with the idea of bipolar fuzzy sets as a more general form of fuzzy sets in 1994. A bipolar fuzzy set is an addition to Zadeh's fuzzy set theory.Talebi and et al studied neighborhood connectivty index and Notion of Complex Pythagoreanof a Fuzzy Graph with Properties and Application [17, 31] and bipolar fuzzy graphs [32, 33]. Its inclusion degree range is [-1, 1]. If an element in a bipolar fuzzy set has a membership degree of 0, it doesn't matter what the corresponding property is. If an element has a membership degree between (0 ,1], it somewhat meets the property, and if it has a membership degree between [-1 ,0) it somewhat meets the implicit counter-property. TIs are molecular descriptors that are found by computing the molecular graph of a chemical substance. They are used in mathematical chemistry, molecular topology, and chemical graph theory. In graphs, these TIs are numbers that explain the way the graph is laid out harold Wiener created the Wiener index (WI) in 1947 [38], which is used to find the boiling point of paraffins. The Zagreb index (ZI) was created by Gutman and Trinajstic in 1972 [8] and is a degree-based TI that is used to find the π-electron energy of a paired system. Fortula and Gutman created the Forgotten Topological Index (F-index) in 2015, which is a degree-based topological index that came after this one [7]. Mondal et al. [18, 19] talk about some neighborhood degree-based topological indices. You can look at different kinds of TIs in [5, 6, 9, 24]. The F-index of the subdivision graph and line graph was given by Amin and Nayeem [2].

1.2. Motivation. Topological indices are a very important part of molecular chemistry, chemical graph theory, spectral graph theory, network theory, and other fields. This TI, the Zagreb index (ZI), was created by Gutman and Trinajstic in 1972 [8]. It is a degree-based TI that is used to figure out the π −electron energy of a paired system. After this topological index, Fortula and Gutman created a new degree-based topological index in 2015 called the Forgotten Topological Index (F-index) [7]. They showed that the F-index has higher correlation coefficients than the ZI when it comes to entropy, predictive ability, and the acentric factor. So this TI is very helpful in molecular chemistry. It is also used in spectral graph theory, network theory, and a number of math areas. Those numbers can only be found in crisp graph. Binu et al. [3, 4] just recently, in 2020, presented the connectivity index and the Wiener index in fuzzy graphs and showed how they could be used to help stop illegal immigration. After reading those papers [1, 10, 11, 12, 22, 23], we also looked at some topological indices on fuzzy graphs. The F-index for bipolar fuzzy graph is presented in this paper, along with some interesting results on it.

1.3. Significance and objective of the article. Various topological index of a graph has numerous uses, and there are numerous results accessible for crisp graphs. However in many practical applications it is seen that many situations cannot be modeled using crisp graphs. To address such situations, it is necessary to define topological indices in a bipolar fuzzy graph. This study introduces the concept of F-index for bipolar fuzzy graphs and provides bounds for different types of bipolar fuzzy graphs such as bipolar fuzzy path, bipolar fuzzy cycle, bipolar fuzzy star, and complete bipolar fuzzy graph ($CBFG$). This article demonstrates that $CBFG$ is the largest possible BFG in terms of the F-index for a given set of vertices. The essay concludes by the application of the F- Index to identify the best health center in providing medical services. Where it was chosen four health centers in Dhi Qar province, Al-Fuhoud District, it has the largest number of patients among other health centers.

2. Preliminaries

In this portion, some basic definitions are provided which are essential to develop our results, most of them can be found in [20, 21].

Let X be a universal set. A FS S on X is a mapping $\Omega : X \rightarrow [0, 1]$. Here Ω is called the membership function of the FS S. Generally a FS is denoted by $\mathrm{S} = (\mathrm{X}, \Omega)$.

Definition 2.1. Let X be a non-empty set and be a given finite set. A FG is a triplet $G = (X, \beta, \Omega)$ such that $\beta : X \rightarrow [0, 1]$ and $\Omega : X \times X \rightarrow [0, 1]$ satisfying $\Omega(u, v) \leq \beta(u) \wedge \beta(v), for\ all\ u, v \in X$ where ∧ represents the minimum.

Abbreviation	Meaning
FS	Fuzzy set
FSS	Fuzzy subset
FG	Fuzzy graph
FSG	Fuzzy subgraph
$PFSG$	Partial fuzzy subgraph
$PBFSG$	partial Bipolar fuzzy subgraph
BFG	Bipolar fuzzy graph
$BFSG$	Bipolar fuzzy subgraph
$CBFG$	complete Bipolar fuzzy graph
TI	Topological index
ZI	Zagreb index
$F - index$	Forgotten Topological Index
MV	Membership value

Table 1 The list of abbreviation

The set X is the set of vertices and $E = \{(u, v) : \Omega(u, v) > 0\}$ is the set of edge of the FG. $\beta(u)$ represents the vertex MV of u and $\Omega(u, v)$ represents the edge MV of (u, v) (or simply uv).

Definition 2.2. Let X be a nonempty set. Then we call a mapping $\beta = (\mu_\beta^P, \mu_\beta^N) : X \times X \to [-1, 0] \times [0, 1]$a bipolar fuzzy set on X such that $\mu_\beta^P(u, v) \in [0,1]$ and $\mu_\beta^N(u, v) \in [-1,0]$.

Definition 2.3 A bipolar fuzzy graph with an underlying set X is defined to be triplet $G = (X, \beta, \Omega)$, where $\beta = (\mu_\beta^P, \mu_\beta^N)$ is a bipolar fuzzy set in X and $\Omega = (\mu_\Omega^P, \mu_\Omega^N)$ is a bipolar fuzzy set in $X \times X$ such that $\mu_\Omega^P(u, v) \leq \mu_\beta^P(u) \wedge \mu_\beta^P(v)$ and $\mu_\Omega^N(u, v) \geq \mu_\beta^N(u) \vee \mu_\beta^N(v)$ for all $(u, v) \in X \times X$. We call β the bipolar fuzzy vertex set of G and Ω the bipolar fuzzy edge set of G, respectively.

Definition 2.4. Let $G = (X, \beta, \Omega)$ be a BFG. Then $H = (X', \beta', \Omega')$ is called a $PBFSG$ of the BFG G if $X' \subseteq X$, $\mu_{\beta'}^P(u) \leq \mu_\beta^P(u), \mu_{\Omega'}^P(u, v) \leq \mu_\Omega^P(u, v)$ and $\mu_{\beta'}^N(u) \geq \mu_\beta^N(u), \mu_{\Omega'}^N(u, v) \geq \mu_\Omega^N(u, v)$ for all $u, v \in X'$.If $\mu_{\beta'}^P(u) = \mu_\beta^P(u), \mu_{\beta'}^N(u) = \mu_\beta^N(u)$and $\Omega^{P'}(u, v) \leq \Omega^P(u, v), \Omega^{N'}(u, v) \geq \Omega^P(u, v)$for all $u, v \in X'$ then H is called $BFSG$ of the BFG G. For $u \in X$, we denote G_u for a $BFSG$ $H = (X - \{u\}, \beta', \Omega')$ of the BFG G with $\mu_{\beta'}^P(u) = \mu_{\beta'}^N(u) = 0$, $\mu_\beta^P(v) = \mu_{\beta'}^P(u), \mu_\beta^N(v) = \mu_{\beta'}^N(u), v \neq u$. For $uv \in E$, G_{uv} represents the $BFSG$ $H = (X', \beta', \Omega')$ of G with $\mu_{\Omega'}^P(uv) = \mu_{\Omega'}^N(uv) = 0, \mu_{\Omega'}^P(xy) = \mu_\Omega^P(xy), \mu_{\Omega'}^N(xy) = \mu_\Omega^N(xy), xy \neq uv$.

Definition 2.5. Let $u_0, u_1 \cdots, u_n$ be distinct vertices of a BFG G. Then the sequence of vertices $P(u_0, u_1 \cdots, u_n)$ is called a bipolar path in G if $(\mu_\Omega^P(u_i, u_{i+1}), \mu_\Omega^N(u_i, u_{i+1})) \neq 0$ for $i = 0, 1, \cdots, n - 1$. The bipolar fuzzy path P is called bipolar fuzzy cycle if $\mu_\Omega^P(u_0, u_n) > 0$ or $\mu_\Omega^N(u_0, u_n) < 0$.

Definition 2.6. Let $u_0, u_1 \cdots, u_n$ be the vertex set of a BFG G. Then $G = (X, \beta, \Omega)$ is called a bipolar star if $(\mu_\Omega^P(u_0, u_j), \mu_\Omega^N(u_0, u_j)) \neq 0$ and $(\mu_\Omega^P(u_i, u_j), \mu_\Omega^N(u_i, u_j)) = 0$ for $i, j = 1, 2, \cdots, n$. Here u_0 is called the bipolar center and $u_1 \cdots, u_n$ are called the pendent vertices of the bipolar fuzzy star G.

Definition 2.7. A BFG G is called a $CBFG$ if $\mu_\Omega^P(uv) = \mu_\beta^P(u) \wedge \mu_\beta^P(v)$ and $\mu_\Omega^N(uv) = \mu_\beta^N(u) \vee \mu_\beta^N(v)$ for all $u, v \in X$.

Definition 2.8. Two BFG $G_1 = (X_1, \beta_1, \Omega_1)$ and BFG $G_2 = (X_2, \beta_2, \Omega_2)$ are called isomorphic if there exist a bijective map $h : X_1 \to X_2$ which satisfies the following conditions:

(i) $\mu_{\beta_1}^P(u_1) = \mu_{\beta_2}^P(h(u_1)), \mu_{\beta_1}^N(u_1) = \mu_{\beta_2}^N(h(u_1))$

(ii) $\mu_{\Omega_1}^P(u_1, v_1) = \mu_{\Omega_2}^P(h(u_1), h(v_1)), \mu_{\Omega_1}^N(u_1, v_1) = \mu_{\Omega_2}^N(h(u_1), h(v_1))$

for all $u_1 \in X_1$, $u_1 v_1 \in \Omega_1$.

Definition 2.9. Let $G = (X, \beta, \Omega)$ be a bipolar fuzzy graph and $u \in X$ then degree of u is denoted by $d_G(u) = (d_G^P(u), d_G^N(u))$ or simply $d(u) = (d^P(u), d^N(u))$ and is defined as

$$d(u) = (d^P(u), d^N(u)) = \left(\sum_{v \in X} \mu_\Omega^P(uv), \sum_{v \in X} \mu_\Omega^N(uv)\right).$$

We set $\Delta_P(G) = \vee_{u \in X} d^P(u), \delta_P(G) = \wedge_{u \in X} d^P(u), \Delta_N(G) = \wedge_{u \in X} d^N(u), \delta_N(G) = \vee_{u \in X} d^N(u)$.

The total degree of G is denoted by $T(G)=\left(T^P(G),T^N(G)\right)$ $where$ $T^P(G)=\sum_{u\in \mathrm{X}} d^P(u) = 2\sum_{uv\in \mathrm{E}} \mu_\Omega^P(uv) \leq 2m$, $T^N(G)=\sum_{u\in \mathrm{X}} d^N(u) = 2\sum_{uv\in \mathrm{E}} \mu_\Omega^N(uv) \geq 2(-m) = -2m$ where m is the number of edges.

3. F-index of fuzzy graphs

Gutman et al. [8] introduced the first and second ZI of a crisp graph in 1972.

Definition 3.1. [8] Let $G = (X, \mathrm{E})$ be a crisp graph. Then the first ZI of the graph G is denoted by $M_1(G)$ and is defined by:

$$M_1(G) = \sum_{u\in X} d^2(u).$$

Definition 3.2. [8] Let $G = (X, \mathrm{E})$ be a crisp graph. Then the second ZI of the G is denoted by $M_2(G)$ and is defined by:

$$M_2(G) = \sum_{uv\in \mathrm{E}} d(u)d(v).$$

These two TIs are degree based TIs.

Definition 3.3. [7] Suppose $G = (X, \mathrm{E})$ is a crisp graph. Then F-index of G is denoted by $F(G)$ and is defined by:

$$F(G) = \sum_{u\in X} d^3(u).$$

Those indices are defined in crisp graph only. In 2020, Kalathian et al. [21] studied first and second ZI for a BFG as follows:

Definition 3.4. [12] Suppose $G = (X, \beta, \Omega)$ is a BFG. Then the first ZI of the BFG G is denoted by $M(G)$ and is defined by:

$$M(G) = \left(M^P(G), M^N(G)\right) = \left(\sum_{u\in X} \mu_\beta^P(u)\left(d^P(u)\right)^2, \sum_{u\in X} \mu_\beta^N(u)\left(d^N(u)\right)^2\right).$$

Definition 3.5. [12] Suppose $G = (X, \beta, \Omega)$ is a BFG. Then the second ZI of the BFG G is denoted by $M^*(G)$ and is defined by:

$$M^*(G) = \left(M^{*P}(G), M^{*N}(G)\right) = \left(\sum_{uv\in \mathrm{E}} \mu_\beta^P(u)d^P(u)\mu_\beta^P(v)d^P(v), \sum_{uv\in \mathrm{E}} \mu_\beta^N(u)d^N(u)\mu_\beta^N(v)d^N(v)\right)$$

We define F-index of a BFG as follows:

Definition 3.6. Suppose $G = (X, \beta, \Omega)$ is a BFG. Then F-index of the BFG G is denoted by $\Gamma(G)$ and is defined by:

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u\in X} \left[\mu_\beta^P(u)d^P(u)\right]^3, \sum_{u\in X} \left[\mu_\beta^N(u)d^N(u)\right]^3\right)$$

Theorem 3.1. Suppose G is a BFG with n vertices and m edges. Then

$\Gamma(G) \leq (n^3(T^P)^3, -n^3(T^N)^3)$,

Proof. Using the fact $0 \leq \mu_\beta^P(u) \leq 1$ and $-1 \leq \mu_\beta^N(u) \leq 0$, the following inequality holds:

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u \in X} \left[\mu_\beta^P(u) d^P(u)\right]^3, \sum_{u \in X} \left[\mu_\beta^N(u) d^N(u)\right]^3\right)$$

$$\leq \left(\left[\sum_{u \in X} \mu_\beta^P(u)\right]^3 \left[\sum_{u \in X} d^P(u)\right]^3, \left[\sum_{u \in X} \mu_\beta^N(u)\right]^3 \left[\sum_{u \in X} d^N(u)\right]^3\right)$$

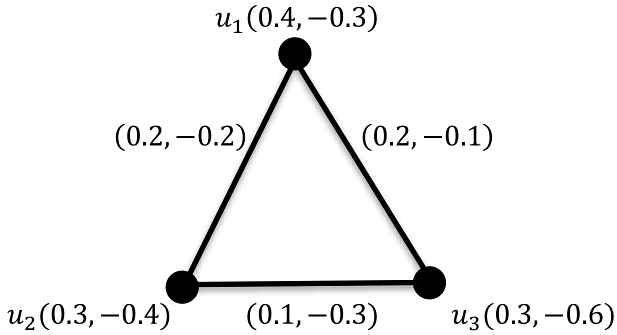
$$\leq ([(n)(T^P)]^3, [(n)(-T^N)]^3)$$

$$\leq (n^3(T^P)^3, -n^3(T^N)^3).$$

Now, the first ZI at a vertex of a BFG is defined below.

Definition 3.7. Let $G = (X, \beta, \Omega)$ be a BFG and $u \in X$. Then the first ZI at the vertex u of the BFG G is indexed by $BZF_1(u\ ;\ G)$ or simply $BZF_1(u)$ and is defined by

$$BZF_1(u) = \Gamma(G) - \Gamma(G_u).$$



$Fig\ 1. A\ BFG\ with\ \Gamma(G) = (0.005554, 0.022553).$

Example 3.1. Suppose $G = (X, \beta, \Omega)$ is a BFG shown in Fig. 1 with vertex set $X = \{u_1, u_2, u_3\}$,and $E = \{u_1u_2, u_1u_3, u_2u_3\}$ such that

	u_1	u_2	u_3
μ_Ω^P	0.4	0.3	0.3
μ_Ω^N	−0.3	−0.4	−0.6

	u_1u_2	u_1u_3	u_2u_3
μ_Ω^P	0.2	0.2	0.1
μ_Ω^N	−0.2	− 0.1	− 0.3

Then the degree is all vertices X

	u_1	u_2	u_3
d^P	0.4	0.3	0.3
d^N	−0.3	−0.5	−0.4

Therefore,

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u\in X}\left[\mu_\beta^P(u)d^P(u)\right]^3, \sum_{u\in X}\left[\mu_\beta^N(u)d^N(u)\right]^3\right)$$

$$= [0.4 \times 0.4, -0.3 \times -0.3]^3 + [0.3 \times 0.3, -0.4 \times -0.5]^3 + [0.3 \times 0.3, -0.6 \times -0.4]^3$$

$$= (0.005554, 0.022553)$$

The next example shows that the F-index of a $BFSG$ is less than the original BFG.

Example 3.2. Let H be a $BFSG$ of the BFG G shown in Fig. 2 obtained by deletion of the edge u_2u_3. Then the degree is all vertices X'

	u_1	u_2	u_3
d_H^P	0.4	0.2	0.2
d_H^N	−0.3	−0.2	−0.1

Therefore, Therefore, F-index of the fuzzy graph H is

$$\Gamma(H) = \left(\Gamma^P(H), \Gamma^N(H)\right) = \left(\sum_{u\in X}\left[\mu_\beta^P(u)d_H^P\right]^3, \sum_{u\in X}\left[\mu_\beta^N(u)d_H^N(u)\right]^3\right)$$

$$= [0.4 \times 0.4, -0.3 \times -0.3]^3 + [0.3 \times 0.2, -0.4 \times -0.2]^3 + [0.3 \times 0.2, -0.6 \times -0.1]^3$$

$$= (0.004528, 0.001457)$$

Note that, from Examples 3.1 and 3.2, we get $\Gamma(H) \le \Gamma(G)$. In the next proposition we proved this fact in general.

Proposition 3.1. Let $H = (X', \beta', \Omega')$ be a $PBFSG$ of a BFG $G = (X, \beta, \Omega)$. Then $\Gamma(H) \le \Gamma(G)$.

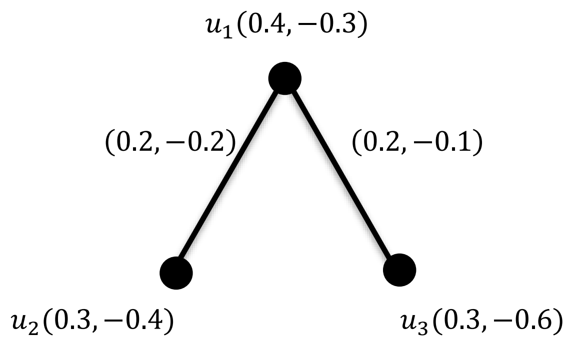


Fig 2. A BFSG of the BFG G in the Fig 1. With $\Gamma(H) \le \Gamma(G)$.

Proof. Since H is a $PBFSG$ of G, then for any $u, v \in X'$, $\mu_{\beta'}^P(u) \le \mu_\beta^P(u), \mu_{\Omega'}^P(u,v) \le \mu_\Omega^P(u,v)$ and $\mu_{\beta'}^N(u) \ge \mu_\beta^N(u), \mu_{\Omega'}^N(u,v) \ge \mu_\Omega^N(u,v)$. So,

$$d_H^P(u) = \sum_{u\in X'} \mu_{\Omega'}^P(u,v)$$

$$\le \sum_{u\in X'} \mu_\Omega^P(u,v)$$

$$\leq \sum_{u \in X} \mu_{\Omega}^{P}(u,v)$$

$$= d_G^P(u).$$

Similarly, we get $\mu_{\beta'}^{N}(u) \geq \mu_{\beta}^{N}(u)$.

Corollary 3.1. Let $H = (X', \beta', \Omega')$ be a $BFSG$ of a $BFG\ G = (X, \beta, \Omega)$. Then $\Gamma(H) \leq \Gamma(G)$. Let $0 \leq t \leq 1, -1 \leq r \leq 0$, the $BFG\ G_{t,r} = (X', \beta', \Omega')$ is a $BFSG$ of the $BFG\ G = (X, \beta, \Omega)$ and is defined as $X' = \{u \in X : \mu_{\beta}^{P}(u) \geq t, \mu_{\beta}^{N}(u) \leq r\}$ and $\left(\mu_{\beta'}^{P}(u), \mu_{\beta'}^{N}(u)\right)\left(\mu_{\beta}^{P}(u), \beta^{N}(u)\right)$,

$\left(\mu_{\Omega'}^{P}(uv), \mu_{\Omega'}^{N}(uv)\right) = \left(\mu_{\Omega}^{P}(uv), \mu_{\Omega}^{N}(uv)\right)$ for $all\ u, v \in X'$.

Theorem 3.2. Suppose G is a BFG and let $0 \leq r_1 \leq r_2 \leq 1\ and\ -1 \leq r_1' \leq r_2' \leq 0$. Then

$$\left(\Gamma\left(G_{(r_2,r_2')}\right) = \left(\Gamma^{P}\left(G_{r_2,r_2'}\right), \Gamma^{N}\left(G_{r_2,r_2'}\right)\right)\right) \leq \left(\Gamma\left(G_{(r_1,r_1')}\right) = \left(\Gamma^{P}\left(G_{r_1,r_1'}\right), \Gamma^{N}\left(G_{r_1,r_1'}\right)\right)\right).$$

Proof. $G_{(r_2,r_2')}$ Is a $PBFSG$ of $G_{(r_1,r_1')}$. Then by Proposition 3.1, the result follows.

Corollary 3.2. Let G be a BFG and let $0 \leq r_1 \leq r_2 \leq \cdots \leq r_n \leq 1\ and\ -1 \leq r_1' \leq r_2' \leq \cdots \leq r_n{}' \leq 0$. Then

$$\Gamma\left(G_{(r_n,r_n')}\right) \leq \Gamma\left(G_{(r_{n-1},r_{n-1}')}\right) \leq \cdots \leq \Gamma\left(G_{(r_1,r_1')}\right).$$

Theorem 3.3. Let $G = (X, \beta, \Omega) = P(u_0, u_1, \cdots, u_n)$ be a bipolar fuzzy path. Then.

$$\Gamma^{P}(G) \leq 2(4n - 3), \Gamma^{N}(G) \leq 2(4n - 3).$$

Proof. Suppose $P(u_0, u_1, \cdots, u_n)$ is a bipolar fuzzy path. Then $d(u_0) = \left(d^{P}(u_0), d^{N}(u_0)\right) = (\mu_{\Omega}^{P}(u_0u_1), \mu_{\Omega}^{N}(u_0u_1)), d(u_n) = \left(d^{P}(u_n), d^{N}(u_n)\right) = (\mu_{\Omega}^{P}(u_{n-1}u_n), \mu_{\Omega}^{N}(u_{n-1}u_n))$ and $d(u_i) = \left(d^{P}(u_i), d^{N}(u_i)\right) = (\mu_{\Omega}^{P}(u_{i-1}u_i), \mu_{\Omega}^{N}(u_{i-1}u_i)) + (\mu_{\Omega}^{P}(u_iu_{i+1}), \mu_{\Omega}^{N}(u_iu_{i+1}))$ for $i = 1, 2, \cdots, n - 1$.Therefore,

$$\Gamma^{P}(P) = \left(\mu_{\beta}^{P}(u_0)\right)^3 \left(\mu_{\Omega}^{P}(u_0u_1)\right)^3 + \sum_{i=1}^{n-1} \left(\mu_{\beta}^{P}(u_i)\right)^3 + \left[\left(\mu_{\Omega}^{P}(u_{i-1}u_i)\right) + \left(\mu_{\Omega}^{P}(u_iu_{i+1})\right)\right]^3$$

$$+ \left(\mu_{\beta}^{P}(u_n)\right)^3 \left(\mu_{\Omega}^{P}(u_{n-1}u_n)\right)^3$$

$$\leq 1 + \sum_{i=1}^{n-1} (2)^3 + 1$$

$$= 2(4n - 3).$$

$$\Gamma^{N}(P) = \left(\mu_{\beta}^{N}(u_0)\right)^3 \left(\mu_{\Omega}^{N}(u_0u_1)\right)^3 + \sum_{i=1}^{n-1} \left(\mu_{\beta}^{N}(u_i)\right)^3 + \left[\left(\mu_{\Omega}^{N}(u_{i-1}u_i)\right) + \left(\mu_{\Omega}^{N}(u_iu_{i+1})\right)\right]^3$$

$$+ \left(\mu_{\beta}^{N}(u_n)\right)^3 \left(\mu_{\Omega}^{N}(u_{n-1}u_n)\right)^3$$

$$\leq 1 + \sum_{i=1}^{n-1} (2)^3 + 1$$

$$= 2(4n - 3).$$

Theorem 3.4. Let $C(u_0, u_1, \cdots, u_n)$ be a bipolar fuzzy cycle. Then

$$\Gamma^P(C) \leq 8(n + 1), \Gamma^N(C) \leq 8(n+1).$$

The proof is similar to the proof of Theorem 3.3.

Theorem 3.5. Let $S(u_0, u_1, \cdots, u_n)$be a bipolar fuzzy star. Then

$$\Gamma^P(S) \leq n^3 + n, \Gamma^N(S) \leq n^3 + n.$$

Proof. As u_0 is the center of the bipolar star S, each u_i is adjacent to u_0, then $d(u_0) = \sum_{i=1}^{n} \left(\mu_\Omega^P(u_0\, u_i), \mu_\Omega^N(u_0\, u_i)\right)$ and u_i is pendent vertex of the bipolar star S for each $i = 1, 2, \cdots, n$, so only u_0 is adjacent to u_i, so, $d(u_i) = \left(\mu_\Omega^P(u_0\, u_i), \mu_\Omega^N(u_0\, u_i)\right)$for $i = 1, 2, \cdots, n$. Therefore,

$$\Gamma(S) = \left(\Gamma^P(S), \Gamma^N(S)\right) = \left(\sum_{u \in X} \left[\mu_\beta^P(u) d_S^P(u)\right]^3, \sum_{u \in X} \left[\mu_\beta^N(u) d_S^N(u)\right]^3\right)$$

$$\Gamma^P(S) = \sum_{u \in X} \left[\mu_\beta^P(u) d_S^P(u)\right]^3$$

$$= \left[\mu_\beta^P(u_0)\, d_S^P(u_0)\right]^3 + \sum_{i=1}^{n} \left[\mu_\beta^P(u_i) d_S^P(u_i)\right]^3$$

$$= \left[\mu_\beta^P(u_0) \left(\sum_{i=1}^{n} \mu_\Omega^P(u_0, u_i)\right)\right]^3 + \left[\sum_{i=1}^{n} \mu_\beta^P(u_i) \mu_\Omega^P(u_0 u_i)\right]^3$$

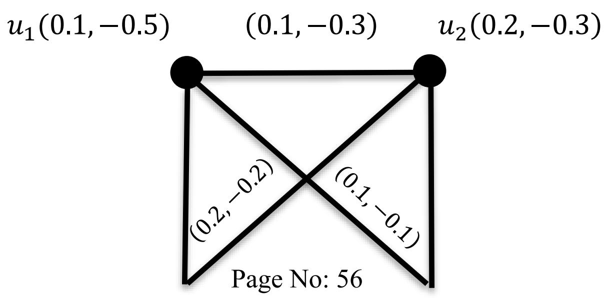
$$\leq n^3 + n$$

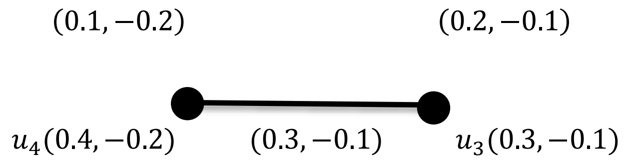
$$\Gamma^N(S) = \sum_{u \in X} \left[\mu_\beta^N(u) d_S^N(u)\right]^3$$

$$= \left[\mu_\beta^N(u_0)\, d_S^N(u_0)\right]^3 + \sum_{i=1}^{n} \left[\mu_\beta^N(u_i) d_S^N(u_i)\right]^3$$

$$= \left[\mu_\beta^N(u_0) \left(\sum_{i=1}^{n} \mu_\Omega^N(u_0, u_i)\right)\right]^3 + \left[\sum_{i=1}^{n} \mu_\beta^N(u_i) \mu_\Omega^N(u_0 u_i)\right]^3$$

$$\leq n^3 + n.$$





$Fig\ 3. A\ CBFG\ with\ \Gamma(G) = (0.020683, 0.033859).$

In the next example, F-index of the $CBFG$ shown in $Fig 3$ is calculated below:

Example 3.3. Let G be a $CBFG$ shown in $Fig\ 3$ with vertex set $X = \{u_1, u_2, u_3, u_4\}$, ,and $E = \{u_1u_2, u_1u_3, u_1\ u_4, u_2u_3, u_2\ u_4, u_3\ u_4\}$ such that,

	u_1	u_2	u_3	u_4
μ_Ω^P	0.1	0.2	0.3	0.4
μ_Ω^N	−0.5	−0.3	−0.1	−0.2

	u_1u_2	u_1u_3	u_1u_4	u_2u_3	u_2u_4	u_3u_4
μ_Ω^P	0.1	0.1	0.1	0.2	0.2	0.3
μ_Ω^N	−0.3	− 0.1	− 0.2	− 0.1	− 0.2	− 0.1

Then the degree is all vertices X

	u_1	u_2	u_3	u_3
d^P	0.3	0.5	0.6	0.6
d^N	−0.6	−0.6	−0.3	−0.5

Therefore, F-index of the fuzzy graph G is

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u\in X}\left[\mu_\beta^P(u)d^P(u)\right]^3, \sum_{u\in X}\left[\mu_\beta^N(u)d^N(u)\right]^3\right)$$

$$= [0.1 \times 0.3, -0.5 \times -0.6]^3 + [0.2 \times 0.5, -0.3 \times -0.6]^3 + [0.3 \times 0.6\ , -0.1 \times -0.3]^3$$

$$+[0.4 \times 0.6\ , -0.2 \times -0.5]^3$$

$$= (0.020683, 0.033859)$$

F-index of a $CBFG$ is studied in the next theorem.

Theorem 3.6. Suppose G is a $CBFG$ with vertex set $= \{u_1, u_2, \cdots, u_n\}$. Then $\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right)$

$$0 \le n(n-1)^3\left(\mu_{\beta_0}^P\right)^6 \le \Gamma^P(G) \le n(n-1)^3\left(\mu_{\beta_n}^P\right)^6 \le n(n-1)^3,$$

$$0 \ge n(n-1)^3\left(\mu_{\beta_0}^N\right)^6 \ge \Gamma^N(G) \ge -n(n-1)^3\left(\mu_{\beta_n}^N\right)^6 \ge -n(n-1)^3,$$

where $\left(\mu_\beta^P, \mu_\beta^N\right)(u_i) = \left(\mu_{\beta_i}^P, \mu_{\beta_i}^N\right)$and $\left(\mu_{\beta_1}^P, \mu_1^N\right) \le \left(\mu_{\beta_2}^P, \mu_{\beta_2}^N\right) \le \cdots \le \left(\mu_{\beta_n}^P, \mu_{\beta_n}^N\right)$.

Proof. As G be a $CBFG$. Then degree of the vertex u_i is

$$d(u_i) = \left(d^P(u_i), d^N(u_i)\right) = (n-i)\left(\mu_{\beta_i}^P, \mu_{\beta_i}^N\right) + \sum_{j=1}^{i-1}\left(\mu_{\beta_j}^P, \mu_{\beta_j}^N\right).$$

Therefore,

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u\in X}\left[\mu_\beta^P(u)d^P(u)\right]^3, \sum_{u\in X}\left[\mu_\beta^N(u)d^N(u)\right]^3\right)$$

$$\Gamma^P(G) = \sum_{u\in X}\left[\mu_\beta^P(u)d^P(u)\right]^3$$

$$= \sum_{i=1}^{n}\left(\mu_{\beta_i}^P\right)^3\left[(n-i)\left(\mu_{\beta_i}^P\right) + \sum_{j=1}^{i-1}\left(\mu_{\beta_j}^P\right)\right]^3$$

$$\leq \sum_{i=1}^{n}\left(\mu_{\beta_n}^P\right)^3\left[(n-i)\left(\mu_{\beta_n}^P\right) + \sum_{j=1}^{i-1}\left(\mu_{\beta_n'}^P\right)\right]^3$$

$$= n(n-1)^3\left(\mu_{\beta_n}^P\right)^6.$$

$$\Gamma^N(G) = \sum_{u\in X}\left[\mu_\beta^N(u)d^N(u)\right]^3$$

$$= \sum_{i=1}^{n}\left(\mu_{\beta_i}^N\right)^3\left[(n-i)\left(\mu_{\beta_i}^N\right) + \sum_{j=1}^{i-1}\left(\mu_{\beta_j}^N\right)\right]^3$$

$$\geq \sum_{i=1}^{n}\left(\mu_{\beta_n}^N\right)^3\left[(n-i)\left(\mu_{\beta_n}^N\right) + \sum_{j=1}^{i-1}\left(\mu_{\beta_n}^N\right)\right]^3$$

$$= -n(n-1)^3\left(\mu_{\beta_n}^N\right)^6.$$

Other inequalities follow similarly.

Let $G = (X, \beta, \Omega)$ be a BFG. Now we construct the $BFG\ C(G) = (X, \beta, \Omega^c)$ with $\mu_{\Omega^c}^P(uv) = \wedge \{\mu_\beta^P(u), \mu_\beta^P(v)\}$ and $\mu_{\Omega^c}^N(uv) = \vee\{\mu_\beta^N(u), \mu_\beta^N(v)\}$ .We call it completion bipolar fuzzy graph of the $BFG\ G$.

Theorem 3.7. Suppose G is a BFG. Then $\Gamma(G) \leq \Gamma(C(G))$.

Proof. As for any $(u,v) \in E, \{\mu_\Omega^P(u,v)\} \leq \wedge\left\{\left(\mu_\beta^P(u)\right), \left(\mu_\beta^P(v)\right)\right\} = \{\mu_{\Omega c}^P(u,v)\}$

$and\ \{\mu_\Omega^N(u,v)\} \geq \vee\left\{\left(\mu_\beta^N(u)\right), \left(\mu_\beta^N(v)\right)\right\} = \{\mu_{\Omega c}^N(u,v)\}$, Therefore, for any $u \in X$,

$$d(u) = \left(d^P(u), d^N(u)\right) = \left(\sum\nolimits_{u\in X}\mu_\Omega^P(u,v), \sum_{u\in X}\mu_\Omega^N(u,v)\right)$$

$$d^P(u) = \sum\nolimits_{u\in X}\mu_\Omega^P(u,v)$$

$$\leq \sum_{u\in X}\mu_{\Omega c}^P(u,v)$$

$$= d_{C(G)}^N(u)$$

$$d^N(u) = \sum\nolimits_{u\in X}\mu_\Omega^N(u,v)$$

$$\geq \sum_{u \in X} \mu_{\Omega c}^{N}(u,v)$$

$$= d_{C(G)}^{N}(u)$$

Now,

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\sum_{u \in X} \left[\mu_\beta^P(u) d^P(u)\right]^3, \sum_{u \in X} \left[\mu_\beta^N(u) d^N(u)\right]^3\right)$$

$$\Gamma^P(G) = \sum_{u \in X} \left[\mu_\beta^P(u) d^P(u)\right]^3$$

$$\leq \sum_{u \in X} \left[\mu_\beta^P(u) d_{C(G)}^P(u)\right]^3$$

$$= \Gamma^P\left(C(G)\right)$$

$$\Gamma^N(G) = \sum_{u \in X} \left[\mu_\beta^N(u) d^N(u)\right]^3$$

$$\geq \sum_{u \in X} \left[\mu_\beta^N(u) d_{C(G)}^N(u)\right]^3$$

$$= \Gamma^N\left(C(G)\right).$$

Corollary 3.3. Among all $n - vertex\ BFG$ with given vertex set, $CBFG$ has maximum $F - index$.

Corollary 3.4. For any $n - vertex\ BFG\ G, then\ \Gamma^P(G) \leq n(n-1)^3$ and $\Gamma^N(G) \geq -n(n-1)^3$.
In the next theorem, $F - index$ is discussed for isomorphic $BFGs$.

Theorem 3.8. Let G_1 and G_2 be isomorphic. Then $\Gamma(G_1) = \left(\Gamma^P(G_1), \Gamma^N(G_1)\right) = \Gamma(G_2) = \left(\Gamma^P(G_2), \Gamma^N(G_2)\right)$.

Proof. As G_1 and $G_2$are isomorphic $BFGs$, there exist a isomorphism φ between G_1 and G_2, i.e. $\varphi : X_1 \to X_2$ is a bijection and for all $u, v \in X_1, \left(\mu_{\beta_1}^P(v), \mu_{\beta_1}^N(v)\right) = \left(\mu_{\beta_2}^P(\varphi(v)), \mu_{\beta_2}^N(\varphi(v))\right)$ and $\left(\mu_{\Omega_1}^P(u,v), \mu_{\Omega_1}^N(u,v)\right) = \left(\mu_{\Omega_2}^P(\varphi(u),\varphi(v)), \mu_{\Omega_2}^N(\varphi(u),\varphi(v))\right)$. Then

$$d_{G_1}(u) = \left(d_{G_1}^p(u), d_{G_1}^N(u)\right) = \sum_{v \in X_1} \left(\mu_{\Omega_1}^P(u,v), \mu_{\Omega_1}^N(u,v)\right)$$

$$d_{G_1}^p(u) = \sum_{v \in X_1} \mu_{\Omega_1}^P(u,v)$$

$$= \sum_{v \in X_1} \mu_{\Omega_2}^P\left(\varphi(u), \varphi(v)\right)$$

$$= \sum_{\varphi(v) \in X_2} \mu_{\Omega_2}^P\left(\varphi(u), \varphi(v)\right)$$

$$= d_{G_2}^p\left(\varphi(u)\right).$$

$$d_{G_1}^{N}(u) = \sum_{v\in X_1} \left(\mu_{\Omega_1}^{N}(u,v) \right)$$
$$= \sum_{v\in X_1} \mu_{\Omega_2}^{P}(\varphi(u), \varphi(v))$$
$$= \sum_{\varphi(v)\in X_2} \mu_{\Omega_2}^{N}(\varphi(u), \varphi(v))$$
$$= d_{G_2}^{N}\big(\varphi(u)\big).$$

Therefore

$$\Gamma^P(G_1) = \sum_{u\in X_1} \left[\mu_\beta^P(u) d^P(u)\right]^3$$
$$= \sum_{u\in X_1} \left[\mu_{\beta_2}^P\big(\varphi(u)\big) d_{G_2}^P\big(\varphi(u)\big)\right]^3$$
$$= \sum_{\varphi(u)\in X_2} \left[\mu_{\beta_2}^P\big(\varphi(u)\big) d_{G_2}^P\big(\varphi(u)\big)\right]^3$$
$$= \Gamma^P(G_2).$$

$$\Gamma^N(G_1) = \sum_{u\in X_1} \left[\mu_\beta^N(u) d^N(u)\right]^3$$
$$= \sum_{u\in X_1} \left[\mu_{\beta_2}^N\big(\varphi(u)\big) d_{G_2}^N\big(\varphi(u)\big)\right]^3$$
$$= \sum_{\varphi(u)\in X_2} \left[\mu_{\beta_2}^N\big(\varphi(u)\big) d_{G_2}^N\big(\varphi(u)\big)\right]^3$$
$$= \Gamma^N(G_2)$$

Hence the result. In the next theorem, bounds for F-index is provided.

Theorem 3.9. For girth a $BFG\ G = (X, \beta, \Omega)$,then $\frac{n\delta_P^6}{m^3} \leq \Gamma^P(G) \leq n\Delta_P^3$ and $\frac{n\delta_N^6}{m^3} \leq \Gamma^N(G) \leq - n\Delta_N^3$.

Proof. Now for $u \in X$, $\delta_P \leq d^P(u) \leq m\left(\mu_\beta^P(u)\right)$ and $\delta_N \geq d^N(u) \geq m\left(\mu_\beta^N(u)\right)$.Therefore ,$\mu_\beta^P(u) \geq \frac{\delta_P}{m}$ and $\mu_\beta^N(u) \leq \frac{\delta_N}{m}$. Now,

$$\Gamma^P(G) = \sum_{u\in X} \left[\mu_\beta^P(u) d^P(u)\right]^3$$
$$\leq \sum_{u\in X} \left[\mu_\beta^P(u)\Delta_P\right]^3$$
$$\leq n\Delta_P^3$$

$$\Gamma^N(G) = \sum_{u\in X} \left[\mu_\beta^N(u) d^N(u)\right]^3$$

$$\leq \sum_{u \in X} \left[\mu_\beta^N(u)\Delta_N\right]^3$$

$$\leq -n\Delta_N^3.$$

Again,

$$\Gamma^P(G) = \sum_{u \in X} \left[\mu_\beta^P(u)d^P(u)\right]^3$$

$$\geq \delta_P^3 \sum_{u \in X} \left[\mu_\beta^P(u)\right]^3$$

$$\geq \delta_P^3 \sum_1^n \left(\frac{\delta_P}{m}\right)^3$$

$$= \frac{n\delta_P^6}{m^3}.$$

$$\Gamma^N(G) = \sum_{u \in X} \left[\mu_\beta^N(u)d^N(u)\right]^3$$

$$\geq \delta_N^3 \sum_{u \in X} \left[\mu_\beta^N(u)\right]^3$$

$$\geq \delta_N^3 \sum_1^n \left(\frac{\delta_N}{m}\right)^3$$

$$= \frac{n\delta_N^6}{m^3}.$$

4. Application of First Entire Zagreb index to identify the most important Health centers.

Health centers are no less important than hospitals in improving public health by providing comprehensive healthcare services to individuals within communities. The importance of health centers goes beyond treating minor illnesses such as influenza, high temperatures, and other injuries; they also focus on preventive care, health education, and disease management to promote the overall well-being of the public. Through their specialized healthcare staff and advanced facilities, health centers contribute significantly to disease prevention, early diagnosis, and timely treatment, reducing the burden of disease on the community. Additionally, health centers work in collaboration with various stakeholders to address healthcare challenges and promote healthy lifestyles. By offering a wide range of medical services, including primary and specialized care, diagnostic procedures, and curative treatments, health centers meet the diverse healthcare needs of individuals of various age groups and backgrounds. Overall, the role of health centers in promoting public health cannot be overstated, as they remain at the forefront of efforts to improve health outcomes and quality of life for entire communities. Therefore, given the importance of four health centers in Dhi Qar province, Al-Fuhoud District, namely, Al-Fuhoud center (Fc), Majri (Mi), Al-Manar (Mr), and Al-Amaira (Aa), we set out to identify the best health center in providing medical services. A health centers vertex membership value can be determined using the following formula:

Vertex MV Positive of a health center $= \frac{\text{Number of patients and dead health center}}{maximum\ \ patients\ and\ dead\ among\ the\ health\ centers}$,

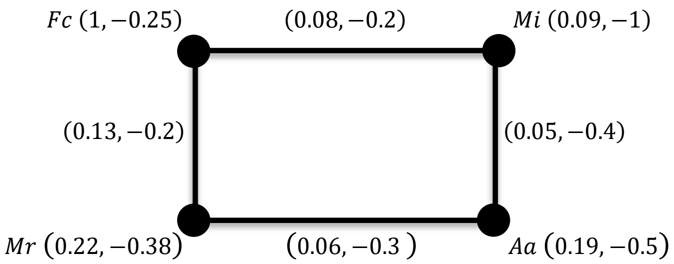
$$\text{Vertex MV negative of a health center} = \frac{\text{Number of dead health center}}{maximum\ \ dead\ among\ the\ health\ centers}$$

All the data are collected from health centers dated eighth month of 2025. The number of patients visiting health centers represents the the score and vertex membership values Positive , while the number of dead represents the score and vertex membership values negative , as shown in Table 2 using the MV law above.

Health centers	patients	MV	dead	MV
Al-Fuhoud center (Fc)	9673	1	2	-0.25
Majri (Mi)	847	0.09	8	-1
Al-Manar (Mr)	2121	0.22	3	-0.38
Al-Amaira (Aa)	1804	0.19	4	-0.5

Table 2. Calculation of vertex membership value

See the bipolar fuzzy graph in the figure 4. The nodes represent the health centers, and the edges is pations between all health centers shown in Table 3.



$Fig\ 4.\ Bipolar\ fuzzy\ graph\ G.$

X	Fc	Mi	Mr	Aa
μ_β^P	1	0.09	0.22	0.19
μ_β^N	-0.25	-1	-0.38	-0.5

E	$Fc\ Mi$	$Fc\ Mr$	$Mi\ Aa$	$Mr\ Aa$
μ_β^P	0.08	0.13	0.05	0.06
μ_β^N	-0.2	-0.2	-0.4	-0.3

Table 3: Weight of nodes in G.

Therefore, the degree is all vertices X

$$d(u) = \left(d^P(u), d^N(u)\right) = \left(\sum_{uv\in X} \mu_\Omega^P(uv), \sum_{uv\in X} \mu_\Omega^N(uv)\right).$$

d_G	Fc	Mi	Mr	Aa
d^P	0.21	0.13	0.19	0.11
d^N	−0.4	−0.6	−0.5	−0.7

Therefore F-index of this BFG is

$$\Gamma(G) = \left(\Gamma^P(G), \Gamma^N(G)\right) = \left(\left[\sum_{u\in X} \mu_\beta^P(u) d_G^P(u)\right]^3, \left[\sum_{u\in X} \mu_\beta^N(u) d_G^N(u)\right]^3\right)$$

$$= (0.009345, 0.250859).$$

Similarly, one can calculate F-index of all vertices. Now the importance of each health center is defined as follows: $S^P(u) = \frac{\Gamma^P(G)-\Gamma^P(G-u)}{\Gamma^P(G)}$, $S^N(u) = \frac{\Gamma^N(G)-\Gamma^N(G-u)}{\Gamma^N(G)}$.Therefore

$$\Gamma^P(G - Fc) = \left[\sum_{u\in X} \mu_\beta^P(u) d_{G-Fc}^P(u)\right]^3$$

$$= [0.09 \times 0.05]^3 + [0.22 \times 0.06]^3 + [0.19 \times 0.11]^3$$

$$= 0.000012,$$

$$S^P(Fc) = \frac{\Gamma^P(G) - \Gamma^P(G - Fc)}{\Gamma^P(G)} = 0.998716,$$

$$\Gamma^N(G - Fc) = \left[\sum_{u\in X} \mu_\beta^N(u) d_{G-Fc}^N(u)\right]^3$$

$$= [-1 \times -0.4]^3 + [-0.38 \times -0.3]^3 + [-0.5 \times -0.7]^3$$

$$= 0.108357,$$

$$S^N(Fc) = \frac{\Gamma^N(G) - \Gamma^N(G - Fc)}{\Gamma^N(G)} = 0.568056,$$

$$\Gamma^P(G - Mi) = \left[\sum_{u\in X} \mu_\beta^P(u) d_{G-Mi}^P(u)\right]^3$$

$$= [1 \times 0.13]^3 + [0.22 \times 0.19]^3 + [0.19 \times 0.06]^3$$

$$= 0.002272,$$

$$S^P(Mi) = \frac{\Gamma^P(G) - \Gamma^P(G - Mi)}{\Gamma^P(G)} = 0.756875,$$

$$\Gamma^N(G_{-Mi}) = \left[\sum_{u\in X} \mu_\beta^N(u) d_{G-Mi}^N(u)\right]^3$$

$$= [-0.25 \times -0.2]^3 + [-0.38 \times -0.5]^3 + [\,-0.5 \times -0.3]^3$$

$$= 0.010359,$$

$$S^N(Mi) = \frac{\Gamma^N(G) - \Gamma^N(G - Mi)}{\Gamma^N(G)} = 0.958706,$$

$$\Gamma^P(G - Mr) = \left[\sum_{u\in X} \mu_\beta^P(u) d_{G-Mr}^P(u)\right]^3$$

$$= [1 \times 0.08]^3 + [0.09 \times 0.13]^3 + [0.19 \times 0.05]^3$$

$$= 0.000514,$$

$$S^P(Mr) = \frac{\Gamma^P(G) - \Gamma^P(G - Mr)}{\Gamma^P(G)} = 0.944997,$$

$$\Gamma^N(G - Mr) = \left[\sum_{u\in X} \mu_\beta^N(u) d_{G-Mr}^N(u)\right]^3$$

$$= [-0.25 \times -0.2]^3 + [-1 \times -0.6]^3 + [\,-0.5 \times -0.4]^3$$

$$= 0.018203,$$

$$S^N(Mr) = \frac{\Gamma^N(G) - \Gamma^N(G - Mr)}{\Gamma^N(G)} = 0.927437,$$

$$\Gamma^P(G - Aa) = \left[\sum_{u\in X} \mu_\beta^P(u) d_{G-Aa}^P(u)\right]^3$$

$$= [1 \times 0.21]^3 + [0.09 \times 0.08]^3 + [0.22 \times 0.13]^3$$

$$= 0.009285,$$

$$S^P(Aa) = \frac{\Gamma^P(G) - \Gamma^P(G - Aa)}{\Gamma^P(G)} = 0.006421,$$

$$\Gamma^N(G - Aa) = \left[\sum_{u\in X} \mu_\beta^N(u) d_{G-Aa}^N(u)\right]^3$$

$$= [-0.25 \times -0.4]^3 + [-1 \times -0.2]^3 + [\,-0.38 \times -0.2]^3$$

$$= 0.009439.$$

$$S^N(Aa) = \frac{\Gamma^N(G) - \Gamma^N(G - Aa)}{\Gamma^N(G)} = 0.962373.$$

As we can see in the calculation above, by removing the Al-Fuhoud center the positive membership degree of the F-index for BFG decreased drastically, so it can be concluded that the Al-Fuhoud center is identify the best health center in providing medical services because, it has the largest number of patients among other health centers.

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