Review Paper on“Fixed Point Theorems in Intuitionistic Fuzzy Metric Space”

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ABSTRACT

In this paper, as a survey paper, “we evaluate many works related to the fixed point theory forthe generalization of fuzzy metric space by using the idea of intuitionistic fuzzy metric space”.

1. “INTRODUCTION” :

Fuzzy set theory was first introduce by Zadeh [1] in 1965to depict the situation in which data is uncertain. Thereafter the concept of fuzzy set was summed up as intuitionistic fuzzy set by K. Atanassov [2] in 1984. Alaca et al. [3] utilizing the possibility of intuitionistic fuzzy sets, they characterized the notion of intuitionistic fuzzy metric space as Park [4] with the assistance of continuous t-norms and continuous t-conorms. The concept of fuzzy metric space was first introduced by Kramosil and Michalek [5] by utilizing the possibility of Intuitionistic fuzzy set, It has an extensive variety of application in the field of computer programming etc. In 2004, Park [6] characterized the notion of intuitionistic fuzzy metric space with the assistance of continuous t-norm and continuous t-conorm. Al-Thagafi and N. Shahzad [7] demonstrated basic settled point theorems for compatible mappings in complete intuitionistic fuzzy metric spaces.Deng [8], Erceg[9],Kaleva and Seikkala [10] and numerous authors gave the concept of fuzzy metric space in different ways.Grabiec[11] introduced the settled point theory in fuzzy metric space.After that Jungck[8] introduced settled point theorem of intuitionistic fuzzy metric space for commuting mappings.The concept of fuzzy 2metric space is given by Sushil Sharma[12] in 2002 and he likewise demonstrated basic settled point theorem in fuzzy 2-metric space.They introduced the concept of non Archimedean intuitionistic fuzzy 3 metric spaces by utilizing the concept of Archimedean fuzzy metric space by Dorel Mihet[13],Sushil Sharma[12] and RenuChugh and Sumitra[14] .Kramosil and Michalek [15] and defined a Hausdorff topology on fuzzy metric space which are often used in current researches. Grabiec [16] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Park [17] also proved some basic results which include Baire's theorem, separability of the space, second countability of the space and it's relation with'separability', 'uniform utmost theorem' etc. "Presently, it remains an important problem in fuzzy topology to obtain an appropriate concept of Intuitionistic fuzzy metric spaces". This problem has been investigated by Saadati and Park [18] wherein they defined precompact sets in Intuitionistic fuzzy metric spaces and proved that any subset of an Intuitionistic fuzzy metric space is compact if and just in the event that it is precompact and complete . Also they defined topologically complete Intuitionistic fuzzy metrizable spaces. "The study of common fixed points of mappings satisfying certain contractive conditions has been at the center of strong research activity and, being the area of the fixed point theory, has very important application in applied mathematics and sciences. It was the defining moment in the "fixed point arena" when the idea of commutativity was used by Jungck to obtain a generalization of Banach's fixed point theorem for a pair of mappings". "This result was further generalized and extended in various ways by many authors. Sessa [19] introduced a weaker version of commutativity for a pair of selfmaps, and it is shown that weakly driving pair of maps in metric space is driving, however the converse may not be true". It is well realized that in the setting of metric space, strict contractive condition don't ensure the existence of common fixed point unless the Space is assumed to be compact or the strict conditions are replaced by stronger conditions. "The study of common fixed points of non-compatible mappings is very interesting. Aamri and El Moutawakil [20] generalized the concept of non compatibility in metric spaces by defining the thought property (E.A.) and proved common fixed point theorems under strict contractive conditions.The property (E. An.) allows replacing the completeness requirement of the space with a more natural state of closeness of the range. A major benefit of property (E.A.) is that it ensures. Sharma and Bamoria [21] elucidated a property (SB) in fuzzy metric spaces for self maps".

2. "PRELIMINARIES" :

2.1 : '<u>SOME BASIC DEFINITIONS USED'</u>:

<u>Definition1:</u>

"A binary operation,$\Diamond$: $[0,1] \times [0,1] \rightarrow [0,1]$,is continuous "t-conorm" if '$\Diamond$' is satisfying the following conditions":

(1) "$\Diamond$ is commutative and associative";

(2) "$\Diamond$ is continuous";

(3) "a $\Diamond$ 0 = a",for all "$a \in [0,1]$"

(4)" a $\Diamond$ b ≤ c $\Diamond$ d", whenever "a ≤ c" and "b ≤ d" for all "a, b, c, d ∈ [0, 1]".

S.NO	AUTHOR	TITLE OF THE PAPER	NAME OF JOURNAL	VOLUME	YEAR	PAGES	MAIN POINTS
1.	“M.Rafi and M.S.M Noorani”	“Fixed point theorem on intuitionistic fuzzy metric space”	“Iranian Journal of Fuzzy Systems”	“3”	“2006”	23-29	“In this paper , authors introducedintuitionistic fuzzy contraction mapping and proved a fixed point theorem in fuzzy metric space.”
2.	“Shams-ur-rahman , M.I. Bhatti, FakharHaider , Muhammad YarBaigand ShabanaAzam”	“Fixed point theorem in intuitionistic fuzzy metric space”	“International Journal of Scientific & Engineering Research”	5	July 2014	1551-1556	“In this paper, authors generalize fuzzy metric space in term of fixed point theorem in Intuitionistic fuzzy metric space.”
3.	“V. Malliga Devi, R. Mohan Raj , and M.Jeyaraman”	“Common Fixed point theorem in intuitionistic fuzzy metric space”	“International Journal of Advanced Engineering Technology”	7	“July-Sept , 2016”	“58-65”	“In this paper, authors proved common fixed point theorems for compatible mappings in complete intuitionistic fuzzy metric”
4.	“NidhiVerma, Dr.RajeshShrivastava”	“A Fixed point theorem in non-archimedean intuitionistic Fuzzy 3-Metric spaces”	“International Journal of Advanced Technology & Engineering Research”	-	2017	112-117	“In this paper,authorsproved a fixed point theorem for a commuting maps and their result generalize and extend some recent results for non Archimedeanintuitionistic fuzzy 3 metric spaces.”

5.	"Akhilesh Jain, Chandel R.S, Hasan Abbas and UdayDolas"	"Common Fixed point theorem in intuitionistic fuzzy metric space under strict contractive conditions"	"International Journal of Recent Scientific Research"	9	July 2018	27716-27721	"In this paper ,authors introduced the notation of Cauchy sequences in an Intuitionistic fuzzy metric space and proved the common fixed point theorem in intutionistic fuzzy metric space under strict contractive conditions."

Definition2:

"Let (X, d) be a 'metric space' and 'A,B,Sand T'be four 'self maps' on X. 'The pairs (A, S) and (B, T) are said to satisfy common property (E.A.)',if there exist'two sequences $\{x_n\}$ and $\{y_n\}$' in X", such that:

$$\text{“}\lim_{n\to\infty} A\,x_n = \lim_{n\to\infty} S\,x_n = \lim_{n\to\infty} \boldsymbol{B}\,x_n = \lim_{n\to\infty} T\,x_n = t\ \forall\ t \in X\text{”}$$

Definition3:

"Let 'Sand T'be two 'self mappings' of an 'Intuitionistic fuzzy metric space' (X, M,N, *, ✧) ,We say that 'Sand Tsatisfy the property (SB)'i.e (Sharma and Bamboria)if thereexists a sequence $\{x_n\}$ in X"such that:

$$\text{“}\lim_{n\to\infty} \mathrm{S}\,x_n = \lim_{n\to\infty} T\,x_n = t\ \forall\ t \in X\text{”}$$

Definition 4:

"Let A and S be maps from an 'intuitionistic fuzzy metric space(X, M,N, *, ✧)'into itself. The maps 'A and S are said to be weakly commuting' ", if,
"$M(ASz, SAz, t) \geq M(Az, Sz, t)\ and\ N(Az, Sz, t)\forall z, X\ and\ t > 0$"

2.2: 'LITERATURE SURVEY' :

2.3: 'Theorem' :

"Let (A,B,C, *, ◊) be an 'intuitionistic fuzzy metric space' with 'ɩ*ɩ≥ɩ' for some 'ɩ ϵ [0,1]' and the condition (FM-6)". "Let P and Q be 'weakly compatible mapping' of A into itself", such that;

(1) "P and Q satisfy the property (S-B)",

(2) “there exist a number K ϵ (0,1)” such that ;

“B(Qx, Qy, kt)>B(Px, Py, t) * B(Px, Qx, t)* B(Py, Qy, t)* B(Py, Qx, t)* B(Px, Qy, t)”

And,

“C(Qx, Qy, kt) <C(Px, Py, t) ◊ C(Px. Qx, t) ◊ C(Py, Qy, t) ◊ C(Py, Qx, t) ◊C(Px, Qy, t)’

‘For all x≠y ϵ A”,

(3) “QA c PA ,ifPA or QA be a close subset of A, then P and Q have a unique common fixed points”.

2.4 :‘Theorem’ :

“Let (X, M, N, *, ◊) be an ‘intuitionistic fuzzy metric space’ with ‘ŧ*ŧ≥ŧ’ for some ŧ ϵ [0,1] and the condition (FM-6)”. “Let A, B , S and T be mapping of X into itself”, such that ;

(1) “AX c TX and BX c SX”,

(2) “(A,S) or (B,T) satisfies the property (S-B)”,

(3) “there exist a number k ϵ (0,1)”, such that;

‘M(Ax, By, kt) ≥ M(Sx, Ty, t) *M(Sx, By, t)* M(Ty, By,t)‘

And,

‘N(Nx, By, kt) ≤ N(Sx, Ty, t) ◊ N (Sx, By, t) ◊ N(Ty, By, t)’, for all x,y ϵ X,

(4) “(A,S) and (B, T) are weakly compatible”,

(5) “One of AX, BX, SX, or TX is a closed subset of X”.

“Then ‘A, B, S and T’ have a unique common fixed point in X”.

2.5 ‘Theorem’ :

“Let (,X ,M ,N,*,◊) be a ‘complete non Archimedean intuitionistic fuzzy 2 metric space’ and ‘q,r:X→X’ be a mapping”, satisfying ;

(i) “r(X)c f (X)”

(ii) “ris Continuous”

(iii) “M(r (x), r (y), a ; αŧ) ≥ M (f (x),f (y), a ; ŧ) v x, y ε

X, 0<α<1”

And, “limɫ M (x, y,a ; ɫ)= 1”

‘N(r (x), r (y),a ; αɫ) ≤ N (f(x),f (y),a ; ɫ)’

for all‘x,yεX’, ‘0<α<1’

And,‘limt̲ N (x, y ,a ; ɫ) = 0’

‘Then q and r have a unique common fixed point’.

2.6 ‘Theorem’ :

“Let (X,M N ,*,◊) be a ‘complete non Archimedean intuitionistic fuzzy 3 metric space’ and

‘m,n :X →X’ be a mapping” satisfying;

(i) “n (X)c f (X)”

(ii) “mis Continuous”

(iii) “ M(n(x), n (y)a,b ; αɫ) ≥ M (f (x), f (y)a,b ; ɫ)”

For all, ‘X,y ε X, 0< α<1’

And, ‘lim ɫ M(x,y,a,b;t)=1’

‘N(n (x), n (y),a,b ; αɫ) ≤ N(m (x), m(y)a ,b ; ɫ)’

For all x,y ε X , ‘0 < α < 1’

And, ‘limɫ N (x,y ,a ,b ; ɫ)= 0’

“Then m and n have a unique common point”.

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